JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 40, No. 1, January–February 2003

Probability of a Laser Illuminating a Space Object

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Laser beam impingement probability is needed to quantify the risk of accidental or unwanted illumination of a space object. A theoretical method of combining laser beam pointing error and space object position error is used to calculate beam impingement probability. The method accounts for position, velocity, and error covariance matrices of laser emitter and space object. Laser slew rate, pointing uncertainty, and beam divergence half-cone angle, as well as space object size and shape, are also included in the formulation. In performing the calculation, parameters are transformed to a laser beam coordinated frame that is centered on the nominal beam axis. Instantaneous and time-dependent impingement probabilities are reduced to one-dimensional contour integrals. The cumulative impingement probability is expressed as a simple integral. A software tool was created to implement the method. Numerical results for example data are presented.

Nomenclature

$\boldsymbol{A}$ = unit vector normal to the laser beam having the largest pointing uncertainty
a = standard deviation of the symmetric probability density, m
$B\pm$ = y-axis intercept parameter in the diagonal frame, m
b = standard deviation along the y axis of the diagonal frame, m
C = satellite and laser combined error covariance matrix in the Earth centered inertial (ECI) frame, m^2
D = laser aperture diameter, m
H = combined hard-body radius, m
$\boldsymbol{L}$ = unit vector in the direction of the laser beam
$M\pm$ = integral limits
$\boldsymbol{N}$ = unit vector normal to the laser beam having the smallest pointing uncertainty
P = rotation matrix from the laser to the diagonal frame
R = distance from the laser emitter to the space object at closest approach, m
S = slope of the space object's trajectory through the diagonal frame
T = ECI to diagonal frame transformation matrix
U = ECI to laser frame transformation matrix
$\boldsymbol{V}$ = velocity of the space object through the diagonal frame, m/s
$\boldsymbol{V}_0$ = relative velocity of the space object with respect to the laser beam at closest approach, m/s
$\boldsymbol{V}_{0L}$ = initial velocity of the laser emitter in ECI, m/s
$\boldsymbol{V}_{0S}$ = initial velocity of the space object in ECI, m/s
W = radius of space object, m
$\boldsymbol{X}$ = initial position of the space object in the diagonal frame, m
$\boldsymbol{X}_0$ = initial relative position of the space object with respect to the laser emitter in ECI, m
$\boldsymbol{X}_{0L}$ = initial position of the laser emitter in ECI, m
$\boldsymbol{X}_{0S}$ = initial position of the space object in ECI, m
x = laser frame abscissa
y = laser frame ordinate, m
y' = diagonal frame ordinate, m
α = laser pointing error standard deviation along the x axis, m
β = laser pointing error standard deviation along the y axis, m
Δ = y-axis intercept variation in the laser frame, m
δ_C = laser beam cone half-angle, rad
δ_L = displacement of laser due to postion uncertainty, m
δ_P = laser beam's pointing error due to pointing uncertainty, rad
δ_S = displacement of space object, m
θ = contour integration parameter, rad
ρ = probability density, m^{-2}
σ = laser pointing error covariance matrix, rad^2
σ_S = laser illumination error covariance at the space object, m^2
φ = rotation angle to transform from the laser to diagonal frames, rad
ω = laser beam slew rate, rad/s

Introduction

THE possibility of a laser beam accidentally impinging on an operational satellite or space debris must be addressed in conjunction with the development of airborne and space-based laser weapon systems.[1,2] Schemes to remove space debris using high-power lasers must also consider the possibility of illuminating other space objects.[3,4] In addition, the use of laser beams for satellite-to-satellite and satellite-to-ground communication requires planning to avoid interference with other operational satellites.

A geometric method was proposed to prevent satellite illumination by ground-based lasers.[5] The angular keep-out regions of the geometric method may unnecessarily reduce the solid angle available for laser operation. Rather than preventing inadvertent illumination, the current work develops a method to calculate the probability of a laser beam illuminating a space object. Once the probability is known, the tradeoff can be made between risk of illumination vs the cost of operational delays.

The cumulative probability represents the total probability of illumination as the space object passes through the laser's field of view. This probability is based on the state vectors of the space object and laser beam source, as well as the associated error covariance matrices. Uncertainties in beam pointing direction, space object size, laser beam divergence, and slew rate also affect impingement probability and are included in the formulation.

Because the laser may not be operating during the entire time that the satellite passes near the laser's field of view, a method was developed to calculate the illumination probability as a function of time and the instantaneous illumination probability.

A formulation was developed that is well suited for efficient computer implementation. The methodology can be readily extended to satellites of irregular shape. Transformation equations that will facilitate the computer implementation of the algorithm are presented.

Received 5 April 2002; revision received 26 September 2002; accepted for publication 24 October 2002.

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Methodology Overview

The geometrical relationships between laser source and satellite are presented in Fig. 1. Uncertainties in the laser state vector causes its nominal position in inertial space to be displaced. In a similar fashion, the satellite is displaced from its presumed position due to errors in its predicted state vector. For purposes of illustration, the laser is directed at a target object. Pointing uncertainty causes beam axis error. This error represents uncertainty in knowledge of beam axis pointing in inertial space. It includes laser platform attitude uncertainty resulting from inertial navigation system and related errors. The laser beam of interest is contained within a cone defined by a cone half-angle. The cone half-angle includes diffraction effects, jitter, boresight error, wave front distortion, etc. The value of the cone half-angle depends on the sensitivity of the satellite being illuminated. Because the intensity is a decreasing function of a ray's off-axis angle, a more sensitive satellite needs a larger value of cone half-angle.

When the impingement probability is calculated, the space object is propagated to the point of closest approach to the nominal laser beam. A laser beam coordinate system is defined at the point of closest approach with one axis aligned with the direction of the beam. Uncertainty in the pointing direction of the laser beam is represented by a two-dimensional Gaussian probability density in a plane normal to the beam's direction. In Fig. 1, the beam pointing error is δ_P. The position error of the space object at closest approach δ_S is represented by a three-dimensional Gaussian probability density. Likewise, the position error of the laser at closest approach δ_L is represented by a three-dimensional Gaussian probability density. Because δ_L and δ_S are assumed to be uncorrelated, the error covariance matrices of laser and space object can be added to obtain the combined error covariance matrix C. The combined error covariance matrix is transformed to the laser beam coordinate frame. Because only uncertainties normal to the laser beam are relevant to the analysis, the two-dimensional laser pointing error covariance matrix and the combined error covariance matrix are added to obtain the relative error covariance matrix between the laser beam and the space object. A coordinate rotation about the laser beam axis serves to diagonalize the relative error covariance matrix, yielding a two-dimensional Gaussian probability density.

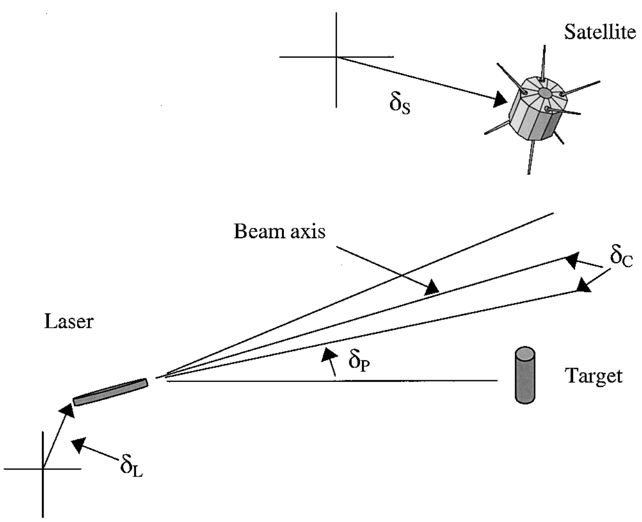

Fig. 1 Geometry of laser beam impingement with error source.

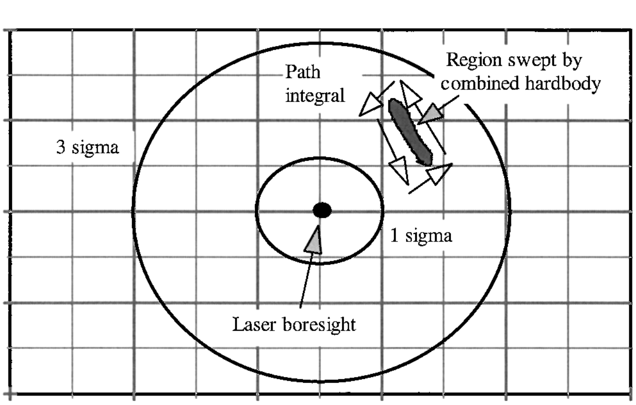

Fig. 2 Region of contour integration in the symmetrized laser frame.

The physical shape of the space object is assumed spherical for consistency with other probability estimation tools. The method will be extended to irregular shape objects in a future work. The hard-body sphere becomes a circle when projected to the two-dimensional diagonal frame. The beam radius at the space object is computed based on the width of the beam at the source aperture, combined with the product of the beam cone half angle δ_C and the range to the object R. The beam radius is added to the hard-body radius to obtain a combined hard-body radius H.

The relative motion of the circle is found by the transformation of the satellite's velocity to the two-dimensional diagonal frame and by accounting for the slew rate of the laser beam. The impingement probability is obtained by integration of the probability density over the area swept out by the combined hard-body circle as the space object traverses the diagonal frame. A scale change is used to make the probability density symmetric. The symmetric form of the probability density permits the area integral to be reduced to a simple one-dimensional contour integral. A similar technique was used in evaluating satellite collision probability.[6] The impingement probability is found by evaluation of the contour integral about the region swept out by the combined hard-body circle, as shown in Fig. 2.

The method was implemented in a computer simulation. Results were obtained using hypothetical data. The methodology can be readily extended to satellites of irregular shape.

Analysis

The laser beam's coordinate system is defined using the direction of the laser beam $\boldsymbol{L}$ and the direction of the axis orthogonal to the beam's direction $\boldsymbol{A}$ that has the largest uncertainty in laser beam pointing. Let $\boldsymbol{N}$ be the unit vector orthogonal to both $\boldsymbol{L}$ and $\boldsymbol{A}$. The transformation from the Earth centered inertial (ECI) frame to the laser frame is given by

$$\boldsymbol{X}_L = U\boldsymbol{X}_{\mathrm{ECI}} \tag{1}$$

$$U = \begin{bmatrix} A(1) & A(2) & A(3) \\ N(1) & N(2) & N(3) \\ -L(1) & -L(2) & -L(3) \end{bmatrix} \tag{2}$$

where $\boldsymbol{A}$, $\boldsymbol{N}$, and $\boldsymbol{L}$ are unit vectors shown in Fig. 3.

The position error covariance matrices of laser and space object are added to form the combined position error covariance matrix C. The combined error covariance matrix is transformed from ECI to the laser beam's coordinate frame in the usual fashion:

$$C_L = UC_{\mathrm{ECI}}U^{-1} \tag{3}$$

If the standard deviations of the laser beam's angular error δ_P has components along the x and y axes given by δ and ε, respectively, then the respective spatial standard deviations at the satellite are α and β, where

$$\alpha = R\delta, \qquad \beta = R\varepsilon \tag{4}$$

The pointing error covariance σ and the associated spatial error

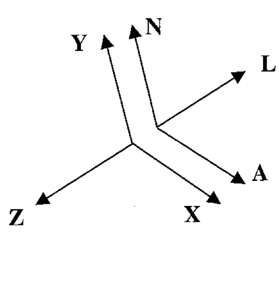

Fig. 3 Laser beam's coordinate system definition with respect to $\boldsymbol{L}$, $\boldsymbol{A}$, and $\boldsymbol{N}$ vectors.

covariance σ_S are given by

$$\sigma = \begin{pmatrix} \delta^2 & 0 \\ 0 & \varepsilon^2 \end{pmatrix} \tag{5}$$

$$\sigma_S = \begin{pmatrix} \alpha^2 & 0 \\ 0 & \beta^2 \end{pmatrix} \tag{6}$$

The combined two-dimensional error covariance matrix becomes

$$C_C = \begin{bmatrix} C_L(1,1) + \alpha^2 & C_L(1,2) \\ C_L(2,1) & C_L(2,2) + \beta^2 \end{bmatrix} \tag{7}$$

The third dimension is not included because it is aligned with the direction of the laser beam.

A rotation ϕ about the z axis serves to diagonalize C_C, yielding

$$C_D = \begin{pmatrix} a^2 & 0 \\ 0 & b^2 \end{pmatrix} \tag{8}$$

The transformation T from ECI to the diagonal frame is given by

$$\boldsymbol{X} = P\boldsymbol{X}_L = PU\boldsymbol{X}_{\text{ECI}} = T\boldsymbol{X}_{\text{ECI}} \tag{9}$$

where

$$P = \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{10}$$

The motion of the space object through the diagonal frame is found by obtaining the relative position and velocity in ECI and then transforming these parameters to the diagonal frame. The relative initial position of the satellite with respect to the laser emitter is given by

$$\boldsymbol{X}_0 = \boldsymbol{X}_{0S} - \boldsymbol{X}_{0L} \tag{11}$$

The relative velocity of the satellite with respect to the laser beam near the closest approach is given by

$$\boldsymbol{V}_0 = \boldsymbol{V}_{0S} - \boldsymbol{V}_{0L} - \omega \times \boldsymbol{LR} \tag{12}$$

Cumulative Probability

Figure 4 illustrates the region in the diagonal frame that is swept out by the satellite's hard body. The lines bounding this region are obtained by transforming $\boldsymbol{X}_0$ and $\boldsymbol{V}_0$ to the diagonal frame:

$$\boldsymbol{X} = T\boldsymbol{X}_0 \tag{13}$$

$$\boldsymbol{V} = T\boldsymbol{V}_0 \tag{14}$$

The boundary lines are given by

$$y = [V(2)/V(1)]x + X(2) - [V(2)/V(1)]X(1) \pm \Delta \tag{15}$$

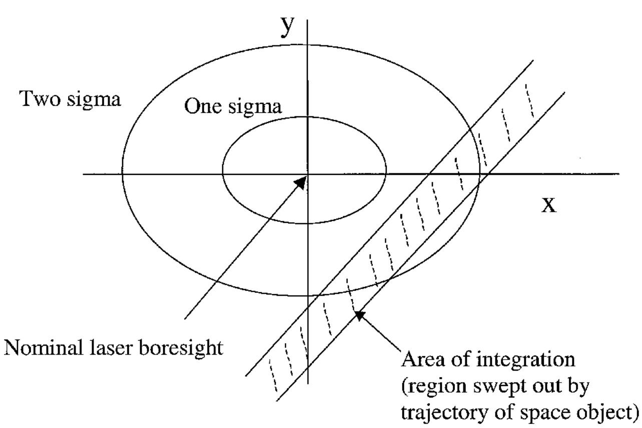

Fig. 4 Integration area in laser frame.

where, for $V(1) \neq 0$, Δ is given by

$$\Delta = H\sqrt{1 + [V(2)/V(1)]^2} \tag{16}$$

If $V(1) = 0$, then the boundary lines are given by

$$x = X(1) \pm H \tag{17}$$

where H is given by

$$H = W + R\delta_C + D/2 \tag{18}$$

The probability of laser beam impingement as the space object traverses the region of the laser beam is found by integrating the probability density over the area bounded by the two lines given in Eq. (15):

$$\text{prob} = \frac{1}{2\pi ab} \iint_{\text{area}} \exp\left(-\frac{x^2}{2a^2} - \frac{y^2}{2b^2}\right) \mathrm{d}x\, \mathrm{d}y \tag{19}$$

A scale change on the y axis makes the probability density symmetric:

$$y' = (a/b)y \tag{20}$$

$$\rho = (1/2\pi ab)\exp[(-x^2 - y'^2)/2a^2] \tag{21}$$

Thus, the two-dimensional integral of the probability density can be reduced to a one-dimensional integral given by

$$\text{prob} = \frac{1}{\sqrt{2\pi}a} \int_{M-}^{M+} \exp\left(-\frac{q^2}{2a^2}\right) \mathrm{d}q \tag{22}$$

where the limits of the integral are given by

$$M\pm = B\pm \Big/ \sqrt{b^2/a^2 + S^2} \tag{23}$$

$$S = V(2)/V(1) \tag{24}$$

$$B\pm = X(2) - SX(1) \pm \Delta \tag{25}$$

This reduction from a two-dimensional integral to a one-dimensional integral is achieved by integrating the probability density in a direction parallel to the direction of the space object's motion (Fig. 4).

Because the probability density is normalized, one obtains a resulting one-dimensional integral given by Eq. (22). Figure 5

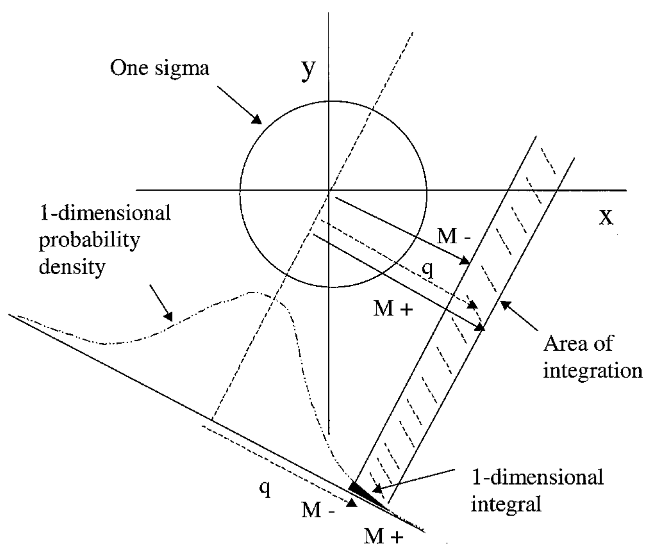

Fig. 5 One-dimensional integral in the symmetrized laser frame.

illustrates the area of integration after a scale change is made to make the probability density symmetric. The integration parameter q, along with the limits $M+$ and $M-$, are also illustrated in Fig. 5.

Time-Dependent Probability

The time that it takes the space vehicle to traverse the laser's field of view may be longer than the time that the laser is active. In this case, the area swept out by the combined hard body during the time at which the laser is active is finite. Figure 2 illustrates such a region in the symmetrized laser reference frame. The probability of illuminating the space vehicle is found by integrating the probability density over the area:

$$\text{prob} = \frac{1}{2\pi a^2}\iint_{\text{area}} \exp\left(\frac{-r^2}{2a^2}\right) r\, \mathrm{d}r\, \mathrm{d}\theta \tag{26}$$

The integration over r can be preformed immediately, yielding

$$\text{prob} = \frac{1}{2\pi}\oint_{\text{perimeter}} \left[1 - \exp\left(\frac{-r^2}{2a^2}\right)\right] \mathrm{d}\theta \tag{27}$$

where the contour integration is about the region swept out by the hard body. If the region of integration excludes the origin, then Eq. (27) becomes

$$\text{prob} = \frac{-1}{2\pi}\oint_{\text{perimeter}} \exp\left(\frac{-r^2}{2a^2}\right) \mathrm{d}\theta \tag{28}$$

If the integration region includes the origin, then Eq. (27) becomes

$$\text{prob} = 1 - \frac{1}{2\pi}\oint_{\text{perimeter}} \exp\left(\frac{-r^2}{2a^2}\right) \mathrm{d}\theta \tag{29}$$

Numerical Example

A computer program was developed to implement the methodology. An example was created with inputs and outputs presented in Tables 1 and 2. The input parameters were chosen to exercise the algorithm and are not operational system values.

The cumulative impingement probability is $8.825e{-2}$, as shown in Table 2. Values of $M+$ and $M-$ are also included for reference. The cumulative probability calculation represents a space vehicle sweep of infinite duration, and so it includes all of the probability.

If the laser is on for a very short duration, such that the space vehicle motion is negligible, the impingement probability is referred to as the instantaneous probability. The instantaneous probability at the point of closest approach is also presented in Table 2.

Table 1 Example case input data

Parameter	X	Y	Z
$\boldsymbol{X}_{0S}$, km	25576.77	33523.47	0.0
$\boldsymbol{X}_{0L}$, km	3869.31	5071.45	0.0
$\boldsymbol{V}_{0S}$, km/s	−2.43786	1.860012	0.0
$\boldsymbol{V}_{0L}$, km/s	−0.36877	0.28136	0.0
$\boldsymbol{\omega}$, rad/s	$-6.848e{-3}$	$5.224e{-3}$	$-7.779e{-3}$
$\boldsymbol{L}$	0.60653	0.79506	0.0
$\boldsymbol{A}$	−0.79506	0.60653	0.0
δ and ε, rad	$3.8785e{-4}$	$2.438e{-4}$	N/A
C, km^2	148.278	7.7664	52.281
	7.7664	31.1745	49.492
	52.281	49.492	100.636

Table 2 Example case output data

Parameter	Value
Beam dispersion, rad	$4.836e{-5}$
Space vehicle radius, km	0.0285
Aperture diameter, m	1
$M+$, km	−3.2687
$M-$, km	−0.5282
Cumulative probability	$8.825e{-2}$
Instantaneous probability at closest approach	$6.7799e{-3}$

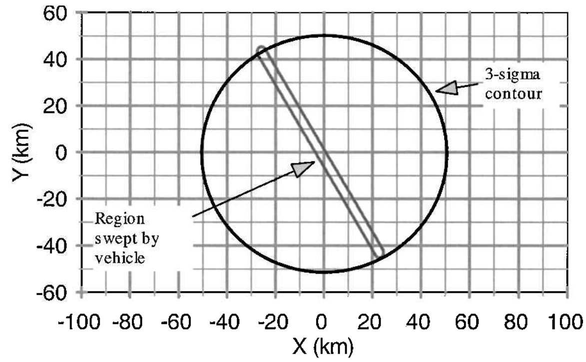

Fig. 6 Space vehicle's sweep region in the symmetrized laser frame.

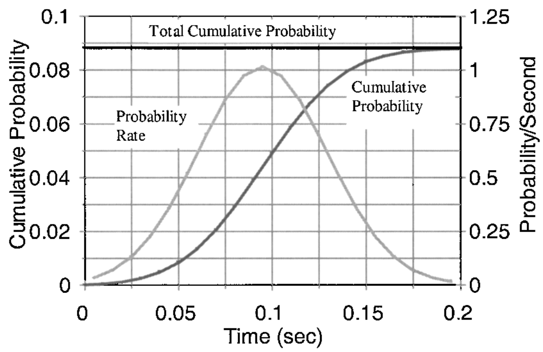

Fig. 7 Impingement probability as a function of time during sweep interval.

If the laser is activated for a finite duration, an impingement area in the symmetrized laser frame defines the space vehicle sweep, as shown in Fig. 6. If the laser beam illuminates any portion of the impingement area, then the space vehicle is considered illuminated. The symmetrized impingement probability density is represented by the 3-sigma contour line in Fig. 6.

As the space vehicle sweeps through the laser's field of view, the impingement probability increases, as shown in Fig. 7. The duration corresponds to a 3-sigma region in Fig. 6 that captures nearly all of the probability. Thus, the total cumulative probability agrees with the time-dependent cumulative probability at 0.2 s, as shown in Fig. 7.

The rate of probability increase is also presented in Fig. 7. The space vehicle begins the sweep at 0.1 s before closest approach and ends the sweep at 0.1 s after closest approach. The 0.2-s duration of sweep captures most of the probability, which is equal to the area under the rate curve in Fig. 7.

One can compute the impingement probability for any time duration within the 0.2-s sweep period. The probability associated with duration from t_1 to t_2 is given by

$$P(t_2 - t_1) = F(t_2) - F(t_1) \tag{30}$$

where F is the cumulative probability as a function of time as shown in Fig. 7.

Conclusions

A general method for calculating the probability of laser beam impingement on a space object was developed. The cumulative probability calculation was reduced to a simple integral. The impingement probability for a finite duration was reduced to the evaluation of a contour integral about a region in the symmetrized laser plane. The impingement probability as a function of time can be used to calculate the impingement probability for an arbitrary duration during the space vehicle's passage through the laser's field of view. The methodology is applicable to satellites having complex geometric shapes. A computer program was developed to implement the technique. An example was created to exercise the algorithms and the computer software. The cumulative impingement probability is in excellent agreement with the time-dependent probability. The impingement probability promises to be a useful measure of risk for laser beam operation in space.

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